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Human decision-making in a non-ergodic additive environment

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Ergodicity Economics presents an intriguing perspective on the validity of the ergodic hypothesis within economic models and the influence of ergodicity breaking on human decision-making processes. Prior research has illuminated the impact of ergodicity breaking within multiplicative settings. However, the implications of ergodicity breaking within additive dynamics, especially in situations carrying a 'risk of ruin' or a complete loss, remain largely unexplored.

In our research, we introduce the concept of 'risk of ruin' into our decision-making model to examine the effects of non-ergodicity in additive dynamics. Our theoretical framework and experiments show that human decision-makers are sensitive to non-ergodicity within purely additive dynamics. This sensitivity manifests itself in significantly different levels of risk aversion depending on the distance and associated likelihood of ruin. These findings underscore the critical role of time averages in human decision-making, suggesting that humans are less irrational than conventionally assumed in behavioural models rooted in expected values.

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Drawing on evidence from Ergodicity Economics, incorporating non-ergodicity has the potential to illuminate common trends in decision-making within compounding systems, like multiplicative growth dynamics. Our research underscores a similar potential for understanding decision-making patterns within additive dynamics when the risk of ruin is present.

1. Introduction

When considering the rationality of human decision-makers, a new wind presents itself in the work of ergodicity economics, pioneered by Ole Peters. In his paper "The Ergodicity Problem In Economics," [1] Peters argues that utility-based decision-making approaches neglect the ergodic hypothesis by overly relying on ensemble averages, i.e. expected values. This overreliance on the expected value causes utility theory to paint human decision-makers as more irrational than appropriate. To illustrate (non-)ergodicity, Peters highlights the contrast between multiplicative (Geometric Brownian motion) and additive (Brownian motion) wealth dynamics. The wealth increments in the additive dynamic conform to the ergodic hypothesis, while the wealth increments in the multiplicative dynamic violate it, causing the time average growth rate to differ from the expected growth rate.

Peters postulates that in a non-ergodic world, such as the multiplicative dynamic, predictions based on time averages approximate human decision-making better than ensemble averages and, consequently, are more appropriate as a measure of rational decision-making. This hypothesis finds empirical support in [2], and [3], wherein intuitive human decision-makers demonstrate a sensitivity for ergodicity breaking within the multiplicative dynamic and behave more rationally than if judged from an ensemble average point of view. Aside from human decision-making, the impact of non-ergodicity can be observed in a large spectrum of fields. Controlling for non-ergodicity explains stable cooperation in humans [4] and organisms [5], sheds light on optimal leverage [6], failings of the GDP [7], social mobility [8], and inequality [9], and can enhance Reinforcement Learning techniques [10].

The type of ergodicity breaking described in [1] and most studied within ergodicity economics stems from the compounding nature of the multiplicative dynamic. However, this is not the only type of ergodicity-breaking human decision-makers come into contact with. Another way of injecting non-ergodicity into a system is by introducing an absorbing barrier, such as ruin. Here ruin refers to an absorbing state from which, once experienced, there is no escape from the lasting negative conclusion, think russian roulette or a ski accident with lasting consequences [11]. Ruin as an absorbing state is not a new concept. It is the main topic of interest for both Ruin Theory [12] and Gambler's Ruin [13]. Nevertheless, little has been done to integrate ruin into the mainstream corpus of economic theory on human decision-making. Two notable examples that incorporate ruin into economic theory are [14], where the focus lies on different company strategies, and [15], who propose ordering preferences based on a desire to survive. As an illustration of ruin and its consequences, consider the following wager. We will flip a coin 20 times, and each time it lands tails, you lose 500 euros, and each time it shows heads, you win 500 euros. Would you take this wager? What if you had a mere 500 euros to your name and no way out of bankruptcy (i.e., ruin)? The proposed bet is an example of the additive-ruin dynamic, which is the primary focus of this paper. In the additive-ruin dynamic, wealth increases and decreases by fixed increments as long as ruin has not been encountered. If we evaluate this wager from the perspective of ensemble averages, the change in starting capital brings little change. Your expected change in wealth is zero, regardless of how much money you possess when encountering the bet. Consequently, a rational decision-maker should be indifferent towards taking the gamble. However, this does not appear to align well with the decisions of actual humans [16] [17] [18], who appear averse towards taking wagers of this type. Classical behavioural economists bridge empirical and theoretical decision-making under uncertainty by introducing a new parameter to be optimised: *utility*. The resulting idiosyncratic utility curve is often characterised by decreasing marginal utility and risk aversion [19]. In other words, as mentioned above, the bet would not be considered attractive because the disutility of losing 500 euros outweighs the utility of gaining 500 euros. This asymmetry is then attributed to psychological and personal biases. These biases cause the expected utility to be negative, which then causes us to veer off the mathematically optimal decision.

Suppose now we were to evaluate this wager and its result on wealth from the angle of time averages. Per Ergodicity Economics [1] [20], a starting point is to investigate potential differences between the expected growth rate and the time average growth rate after the introduction of the absorbing ruin barrier. The introduction of an absorbing barrier does not impact the expected growth rate. Surprisingly, the time-average growth rate, commonly used in ergodicity economics, also remains unaffected in a driftless additive environment. As such, humans should be indifferent between the two bets despite the absorbing barrier, which is the same observation Ole Peters [1] makes when absorbing barriers are absent. However, if we investigate the time average and expected value of your actual wealth, a new feature of the dynamic appears. In that case, ruin prevents those who have experienced it from returning to the ensemble average. As such, the time average and the ensemble average drift apart, making wealth and its dynamic non-ergodic.

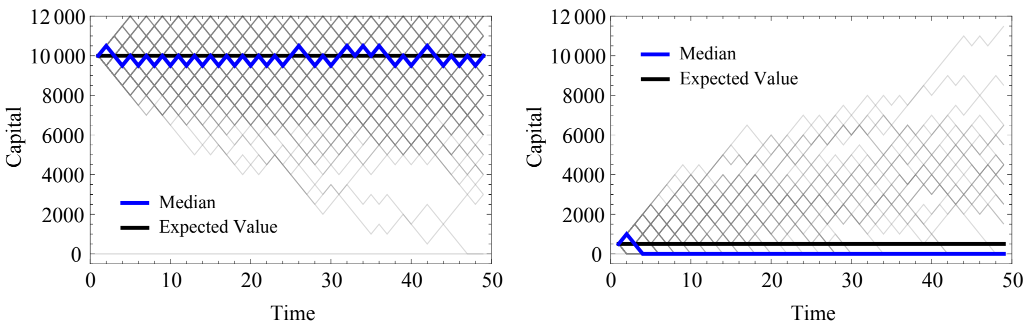

Figure 1: The left panel illustrates the capital of 200 simulated agents starting with 10 000 euros playing a symmetrical bet for which they can gain 500 euros or lose 500 euros for 50 repetitions or until they encounter ruin. The right panel describes 200 agents playing the same bet, but now the agents start with a capital of 500 euros. The black lines depict individual tracks, and the blue line represents the median agent in each dynamic. Observing the median agent in both figures shows that for our time frame, not taking the bet is decidedly not irrational when you start with 500 euros

The gamble's expected value remains zero, equivalent to letting $n \to \infty$ individuals play this gamble and averaging over the obtained n outcomes. In contrast, one person in the long-time horizon ($t \to \infty$) will have been absorbed with probability one into the state of ruin, regardless of their starting wealth, as in accordance with the Gambler's Ruin problem for fair bets [13]. However, the certainty of experiencing ruin regardless of the magnitude of the starting capital wealth only emerges at the limit $t \to \infty$, as starting capital significantly impacts the probability of encountering ruin for bets with a finite number of repetitions as shown in Figure 1. Consequently, if we consider time averages, the reason for not taking the bet has little to do with personal biases and heuristics but rather the mathematical properties of the dynamic, such as the number of repetitions and starting wealth. Within this paper, we take a closer look at whether human decision-makers are sensitive to ergodicity breaking due to ruin and consequently oppose the predictions from the ensemble average based approaches of classical behavioural economics. Note that the precise capital at which the absorbing barrier is defined is of little importance. The important feature of the barrier is its distance with respect to starting capital 2(b). As such, the absorbing barrier chosen in our illustration and experiment, 'zero', holds no meaning apart from facilitating a clear explanation. This is different from the special value zero holds within the multiplicative dynamics, typically studied within the field of ergodicity economics. Here zero represents the limit behaviour ($t \to \infty$) of a downward drifting Geometric Brownian Motion.

One way of modelling human decision-making in the additive ruin dynamic has been explored in [20]. The authors propose an ergodic transform for the additive-ruin wealth dynamic by including the option of borrowing capital. In [20], borrowing, in turn, leads to a wealth regime consisting of three distinct regions: The first region is when wealth is positive; The second region when wealth is negative, but loans can be paid back; The third region when wealth is very negative, and the positive wealth dynamic is no longer sufficient to reimburse loans. The last scenario is equivalent to having a loan for which your wage cannot cover the monthly interest cost, and consequently, you cannot pay back the principal. These three scenarios lead to different approaches towards risk, ranging from risk-neutral when wealth is positive to increasingly risk-averse when wealth is negative. Giving a Linex utility function or its more extreme cousin in which risk aversion grows asymptotically to infinity.

Another approach is to focus on the probability of experiencing ruin, given a wealth dynamic and the proximity to ruin (i.e., the relative starting wealth). Unlike the approach proposed by Raposo et al. [20], this approach requires no borrowing and is closer to work on Ruin Theory [12]. The principal difference between Ruin Theory and the approach employed within this paper lies in the time scales of interest. Where ruin theory is primarily interested in the risk of eventual ruin, we focus on the probability of ruin within a given timeframe. To model the probability of ruin, we use both analytical models derived from Dyck-Paths 4(a) and extensively use Monte Carlo Methods when at the current limits of analytical modelling, a common approach in physics-driven investigations into human decision-making [21–24].

This paper primarily focuses on the strategy that avoids ruin by evading excessive risk-taking. This is the dominant strategy allowed to respondents who operate in their own vacuum. However, when decision-makers are allowed to interact, another strategy appears in the form of cooperation. This strategy increases the complexity of the experienced dynamic, requiring an extensive inquiry on the interplay between agents, where collective risk [23–25], rewards and penalties [22], group structure [21] add layers of complexity. This increase in complexity highlights the importance of understanding the ergodicity of these dynamics [5]. Consequently, this manuscript seeks to contribute to the broader discourse on human decision-making within complex systems. We intend the insights presented herein to serve as foundational elements for subsequent explorations into the nuanced dynamics that facilitate interactions among decision-makers.

(a) the aim of this manuscript

We set out to investigate the applicability of the human decision-making paradigm proposed by ergodicity economics by examining the presence of a sensitivity towards ergodicity breaking in the additive ruin dynamic. Our work continues on the empirical work performed in [2] and [3] by introducing human decision-makers to a different type of ergodicity-breaking, additive-ruin.

The task and experiment presented to respondents align with the experimental setup as in [3], which charges respondents with indicating their favoured bet within a bet couple. Furthermore, analogous to [3], we do not assume a particular utility function and distinguish between risk-loving and risk-averse choices by equalling the expected value within each bet couple but differing the variance in outcomes. Four critical changes were made to adapt the experimental setup to the new dynamic.

Firstly, respondents were actively discouraged from encountering ruin i.e., hitting the absorbing barrier at zero. At the start of the experiment, respondents were informed that encountering ruin at any point in the experiment would automatically exclude them from receiving bonus payments, ensuring that hitting the absorbing barrier had a tangible and negative impact. However, encountering ruin did not stop the stated preference experiment, nor did we inform respondents if and when they encountered ruin. We opted for this approach to maximise the possible comparison between respondents with small starting capitals and those with large starting capitals. By design, respondents who started off with the largest starting wealth 10 000 euros could only encounter ruin rarely, regardless of their choices within the experiment. At the same time, it could be commonplace for those starting with 1000 euros, depending on their choices. Next, we informed respondents that they would receive no update on their capital throughout the experiment. We did not inform respondents about the variability in starting capitals. Lastly, we checked their understanding of the setup by double-checking whether they internalised their starting capital and the consequence of encountering the absorbing barrier.

Secondly, all bets were designed to have an expected value of zero or slightly above or below zero. Consequently, ruin was a consequence of the experienced variance rather than a downward-sloping expected value over bets. This deliberate choice was made to prevent respondents from feeling too discouraged about their prospects, as this could result in the sense of helplessness in avoiding ruin [26] [27]. Such a feeling could, in turn, undermine the perceived significance of their choices. [28]

Thirdly, at the onset of the experiment, respondents were randomly assigned one of three starting capitals. Each of these starting capitals implied different probabilities of ruin, ranging from no risk of ruin, no matter the chosen bets, to encountering ruin 67.7% of the time when solely picking the risky option. The equalised expected values, allowed us to solely concentrate on the effect of starting capital on the likelihood of ruin and the resulting attitudes towards risk.

Finally, we pulled respondents from a large non-homogenous population and further increased the respondent size (n=400). Diversifying the respondent base meets one of the limitations of [3], which used a convenience sample of economic students (n=81). To keep attention high, we reduced the amount of bet couples presented to each respondent to 35.

2. Methods

To reach a large and varied pool of respondents, we used the services provided by Prolific, an online participant recruitment platform. All participants were assigned one of three starting capitals and asked to indicate their preferred bet within 35 bet couples. This amount of bets is slightly reduced compared to the previous experiment [3] to keep attention high. The upcoming methodology will be organised into four sections. Initially, we will examine the process of generating bets. Subsequently, we will delve into the technique employed to estimate the risk of ruin and assignment of starting capitals. The third section will focus on the sampling procedure. Finally, the fourth section will revolve around the data analysis. Before implementation, the Ethics Committee for Human Sciences of the Vrije Universiteit Brussel (ECHW) approved all experimental procedures.

(a) Creating bet pairs

As indicated earlier, to avoid the need for fitting a utility function, we use a setup similar to [3] in that we create bet couples in which both bets have an equal expected value but differing variance.

This setup allows us to unambiguously label one bet as the riskier option within a bet couple. To keep the risk of each bet couple relatively similar and the magnitude of outcomes within the range of those used in [3], all bet couples consisted of a bet where one can gain and lose around 500 euros on the one hand and a bet with the gain and loss around 200 euros on the other hand. To ensure diversity in bet couples, outcomes were allowed to vary 5% from these reference values as long as both bets kept their expected value equal. Figure 2 sets out all bet couples that respondents were offered. In addition, each participant was presented with a set of five obvious bets as a control measure, as well as three attention checks. To be eligible for inclusion in the subsequent analysis, respondents must accurately identify at least three of the five control bets and fail none of the three attention checks. Of the initial 400 participants, 378 met these selection criteria. A comprehensive list of all bets and attention checks is provided in Appendix 4(b).

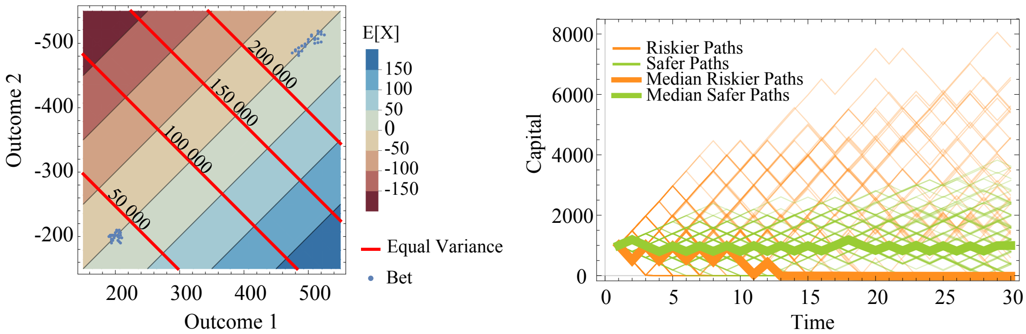

Figure 2: The left graph illustrates all 30 bet couples presented to respondents and their expected value and variance. The bottom left cluster of blue dots represents the bets with outcomes around gaining and losing 200 euros. The top right cluster represents bets with a magnitude of 500, all with a maximum 5% deviation from the aforementioned reference values. The red lines are contour lines on which the level of variance remains equal. Variance increases from the left bottom corner towards the top right corner. The right graph shows the evolution of wealth, starting at 3000 euros, when purely picking either the safe or risky option.

(b) Modelling the risk of ruin and assigning starting capitals

We used an analytical formula derived from the mathematical branch concerned with Dyck-Paths [29] and its extensions to model the risk of ruin. This analytical approach accelerated the modelling step of the experiment. This approach was further complemented by Monte Carlo simulation, which confirmed the validity of our analytical approach and enabled us to refine the desired dynamics even further.

The analytical approach originating in the field of Dyck-Paths yields the following formula $\sum_{n=0}^{t/2-1} \frac{A_n(2,r)}{2^{2n+r}}$ with $r = \left\lceil \frac{K}{\|\beta\|} \right\rceil$ rescaled staring wealth, a detailed explanation can be found in appendix 4(a). Having decided on the magnitude of the bet outcomes ($\|\beta\| = \{200, 500\}$) and the number of bets ($t = 30$), we have one free parameter: *starting capital* K. We then used the obtained formula to model the impact of particular starting capitals on the risk of ruin. Through this procedure, we selected 1000 euros, 3000 euros, and 10 000 euros as starting capital, all of which carry varying degrees of risk of ruin, dependent on their distance (r) to the absorbing barrier. Figure 3 gives an overview of the probabilities of encountering ruin for a given starting capital when always employing a pure strategy, i.e., always selecting the safe or risky bet couple, as well as the probabilities of encountering ruin when employing a mixed strategy.

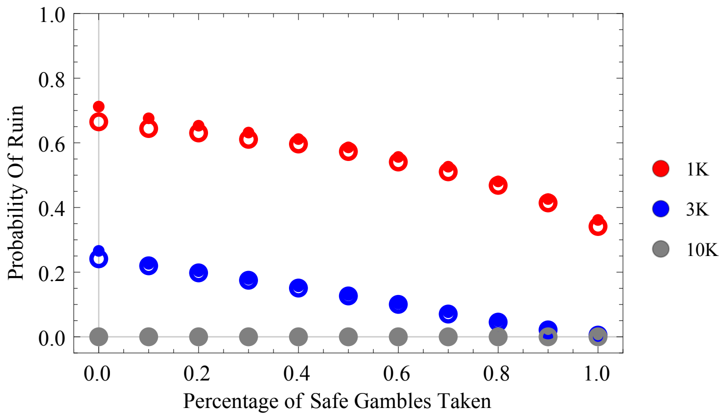

Figure 3: This figure represents the probability of ruin within our experiment for three starting Capitals (1000, 3000, 10 000 euros) when employing pure or mixed strategies. The full dots represent both the theoretical probability of ruin obtained from the Catalan numbers for the pure strategies and the probability of ruin obtained through Monte Carlo analysis for the mixed strategies. The empty dots in the same colour represent the odds of experiencing ruin for the actual bets used within our experiment, which are no longer symmetrical— these were also obtained by Monte Carlo simulation. If only one dot can be observed, this entails that the dynamic resulting from the symmetrical case and the dynamic resulting from bets used within the experiment were identical.

(c) Respondents, implementation, rewards and data analysis

In contrast to [3], respondents were pulled from a sizeable non-homogenous population. We used the online research tool Prolific to target 400 respondents residing in the European Union. Prolific maintains stringent measures to ensure data quality and combat bots, thus providing reliable participant sourcing. Selecting EU residents ensured that respondents had internalized the euro's value, the currency employed throughout the experiment. Before starting the experiment, respondents gave their informed consent.

At the start of the experiment, each respondent was randomly assigned one of three starting capitals (1000 euros, 3000 euros, or 10 000 euros) and randomly selected to either receive time pressure or no time pressure, analogous to [3]. To control for additional stochasticity, which results from one order of bets being more favourable regarding the probability of encountering ruin than another, the order in which bets appeared was kept constant. In addition to their hourly remuneration, we offered five prizes as potential rewards to encourage respondents and minimize the likelihood of excessive risk-taking, often associated with winner-take-all games. We attributed these bonus payments to the five respondents who performed best during the experiment (amassed the most capital). Further, we informed respondents that if they were to experience ruin, they would no longer be considered for these additional prizes. In descending order, the bonus payments were: €125, €100, €75, €50, and €25.

Analogous to [3], we use the curvature argument and model the respondents' choices through a binomial distribution, with probability p of opting for the safe bet. We determine this probability using the simple Bayesian updating method outlined in [3].

3. Results

(a) The effect of time pressure on response time and risk aversion

In contrast to [3], we observe no effect of time pressure on response time and the displayed risk aversion. Median completion times for non-timed and timed respondents were respectively $149s$ and $145s$. A two-sample Kolmogorov-Smirnov test on both distributions yields $p = 0.59$, providing insufficient evidence for a different distribution in completion times. We believe this lack of difference is largely a consequence of the inherent time pressure all respondents experienced due to the payment scheme utilized by Prolific. Respondents on Prolific are paid for the predicted completion time of a survey regardless of how long it takes them to complete it. As such, they already had an inherent incentive to complete the survey faster rather than slower, a stimulus absent in the incentivization scheme employed by [3]. Consequently, the time pressure imposed by us could have been of an additional nature.

(b) Capital evolutions within our experiment

In figure 4, the left graph illustrates the capital evolution for all respondents for the coin flip we used to evaluate final capitals. Unsurprisingly, respondents with 10 000 euros never experienced ruin within the experiment, while some respondents who started with 1000 euros experienced ruin already at step 4. At the end of the experiment, the ruin-state absorbed 24% of the respondents who started with 1000 euros, alluding to the predominately risk-averse behaviour displayed by these respondents. These outcomes result from one coin flip used to determine the highest-performing respondents. To generalise these findings, we kept respondents' choices equal and performed a Monte Carlo simulation on coin flips. The right graph of figure 4 shows the results of this Monte Carlo simulation. The median proportion of participants starting with 1000 euros who encounter financial ruin is 43%. For those beginning with 3000 euros, the median is 4%; for those with 10 000 euros, the median percentage of individuals experiencing ruin is 0%. These percentages align with the mixed strategies modelled in figure 3. Corresponding to mixed strategies heavily favouring risk-averse bets for those starting off with 3000 and 1000 or indicating a low fraction of risk-seeking participants in those starting capitals .

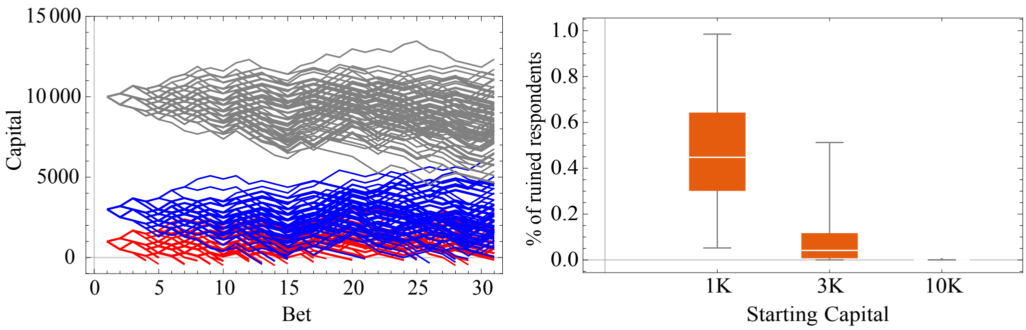

Figure 4: The graph on the left illustrates the capital evolution for all respondents for the coin flip we used to evaluate final capitals. The red lines represent respondents who started off with 1000 euros. Blue is those who had 3000 euros, and the grey lines are the respondents starting with 10 000 euros. A visualisation of 1000 different coin flip scenarios can be found in the boxplots on the right.

(c) The effect of proximity to ruin on risk aversion

We further generalise the previous finding by estimating the likelihood of selecting the safe bet given a starting capital for each bet couple. We then verified the normality for the obtained posterior distributions for all bet couples by the Shapiro-Wilk test. This normality allowed us to generalise our findings employing propagation of uncertainties through which we estimated the tendency of choosing the safe bet option when assigned a particular starting capital. Upon initial observation, we found that all respondents were highly risk averse, much more so than respondents in [3].

Nevertheless, different groups of respondents show different levels of risk aversion. A one-sided z-test reveals that the group nearest to ruin exhibited a significantly ($p = 0.001$ and $p = 4\text{x}10^{-5}$) more conservative approach than their peers, choosing the safe bet option $P = 73.1 \pm 0.7\%$ of the time. The group with a starting capital of 3000 euros displayed a slightly greater inclination towards risk aversion, than their 10k counterparts, albeit without reaching a statistically significant level, they opted for the safe bet $P = 69.9 \pm 0.8\%$ and $P = 68.9 \pm 0.8\%$ of the time respectively. These different levels of risk aversion detailed in figure 4 provide evidence that the prospect of ruin had a discernible influence on the decision-making processes of all participants, having the most pronounced impact on those closest to the brink of ruin. These findings fit within the paradigm proposed by Ergodicity Economics. Ergodicity Economics informs us that respondents are concerned about their time average rather than the ensemble average. Time averages imply a natural desire to avoid encountering absorbing barriers when they entail undesirable consequences such as 'ruin' or the exclusion from obtaining rewards.

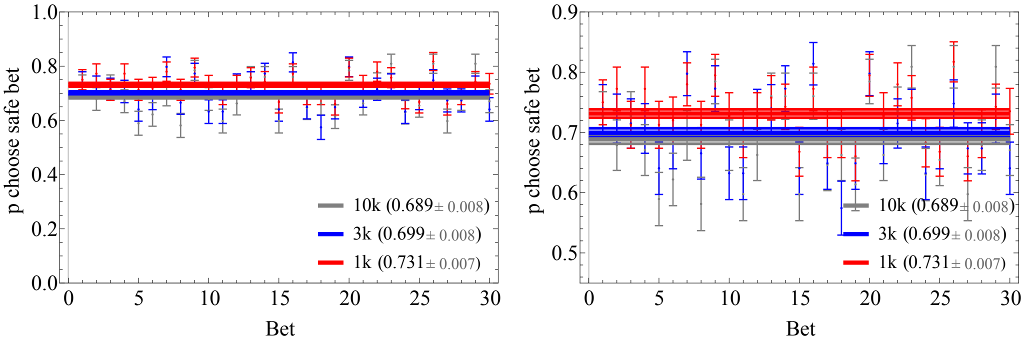

Figure 5: The scatter plot in the left figure displays the estimated probabilities for taking the safer bet for each bet couple as a function of the starting capital. The horizontal error bands indicate the overall estimate of this probability for each starting capital. The right panel provides a zoomed-in view of the same results for more precise visualization.

4. Discussion

The role of ergodicity breaking in the additive ruin dynamic

In 2019 Peters [1] offered a new perspective on behavioural economics by noting that the expected value, which is often used as a base case to model human decision-making, might not be the only or best choice if we live in a non-ergodic world. To illustrate the consequences of non-ergodicity [1] juxtaposes two prevalent wealth dynamics, additive earnings and multiplicative returns. In this illustration, the additive wealth increments represent an ergodic world and the multiplicative wealth increments a non-ergodic world. These settings sparked the investigation on whether human decision-makers are sensitive towards ergodicity breaking. Both [2] and [3] use these contrasting dynamics to observe human behaviour within both ergodic and non-ergodic dynamics and found that humans adapted their behaviour to accommodate the change in dynamic and (non-)ergodicity, strengthening the hypothesis that time average optimisation is a better proxy for human behaviour than expected value optimisation. We expand on these findings and insight by studying human behaviour within a different non-ergodic dynamic additive ruin. This setting follows the general payoffs of the traditional additive dynamic but with an additional absorbing barrier that engenders negative consequences, i.e., 'ruin'. To investigate human behaviour within the additive ruin setting, we used an approach that differs from [2] in two important ways. First, we use significantly fewer bets to reduce potential learning effects [30], which could emerge because of the simplistic nature of the additive ruin setting and combat respondent fatigue [31]. Second, we unambiguously display bet outcomes to circumvent the need for a trial phase in which respondents could internalise the bet outcomes and potentially learn how to play the game, which could cause them to play strategically rather than intuitively. We employed uncertain outcomes for both bets within each couple to combat the potential description-experience gap [32] this methodological choice could have brought about. The most distinct difference with [3] is that this paper addresses some of its shortcomings regarding the respondent base by expanding and diversifying it.

Classical literature on human behaviour often perceives risk aversion as irrational behaviour caused by personal biases and heuristics. However, within our setup, this is not the case. In our experiment, risk-taking behaviour implies that a respondent was more often excluded from the bonus pool than if they played in a risk-averse manner. The irrationality of risk-taking behaviour is doubly true when considering that the bets offered to the respondents have symmetrical payoffs. This symmetry implies that all respondents would have experienced gamblers' ruin if the game had continued without end. Gambler's ruin tells us that ruin is inevitable within our experiment, regardless of starting capital, when $t \to \infty$, with as sole differentiating factor the swiftness with which a respondent could have encountered the absorbing barrier, i.e. 'ruin'. The effect of wealth and the resulting speed of experiencing 'ruin' caused the less fortunate respondents to be more risk-averse than their more wealthy counterparts. This result, while significant, is surprisingly small relative to the large differences in the probabilities of ruin given a starting capital within our experiment, see 2(b).

The small effect could result from the natural limitations of defining ruin within any experimental setup. 'Ruin' within our experiment entailed that the respondents who experienced it were no longer considered for one of the bonus payments. However, regardless of their survival, all respondents were compensated for their participation. Consequently, the type of 'ruin' respondents experienced within our investigation is far from what one might consider actual ruin. While the 'ruin' we imposed on respondents had little impact on compensation, the proximity to 'ruin' significantly affected the decision-making displayed by our respondents. The employed incentivization scheme could also have influenced the low level of risk aversion obtained within our experiment. Considering that respondents need to be among the top 5 performers to receive the bonus payment, which could have made respondents more prone to risk seeking. This effect is in part offset by the higher odds of hitting the absorbing barrier when opting for a risk-seeking strategy. However, all respondents were exposed to the same incentivization scheme. As such, this should not impact the differences in risk aversion, given different starting capitals.

This leaves us with the question of to what extent this experiment demonstrates a distinction between the two competing paradigms, ergodicity economics and expected utility theory. After all, both models can explain the general trend of observations. However, it is illuminating to identify which assumptions must be made in both paradigms. Indeed, this effect of proximity to ruin can be explained using the classical decision-making models grafted on expected values, such as expected utility theory, by making the following deductions. First, we would assume that all respondents had primarily concave utility functions, which leads to the predominantly risk-averse behaviour observed within the experiment. Second, we assume a Laplacian utility function [33] in which concaveness depends on wealth, making them less concave as wealth and, therefore, starting capitals increase. Note that in other interpretations of Expected Utility Theory, utility only depends on wealth changes, regardless of the current wealth level, e.g., the value function employed in prospect theory [16] A third assumption is the non-linear relation of this decreasing concaveness. The second and third assumptions are classically incorporated into diminishing marginal utility. These three assumptions point toward utility functions that display decreasing absolute risk aversion (DARA) [34]. We could then conclude that respondents are risk averse because of idiosyncratic reasons and that the increase in wealth in part offsets the observed loss aversion, the extent of which depends on the individual utility functions.

However, these assumptions follow naturally when trying to optimise the time average within the additive ruin dynamic, which suggests that risk-averse behaviour, based on the starting capital, is a more mathematically suited approach for dealing with additive ruin, which does not need to contrast with the Von Neumann-Morgenstern utility theorem [15]. Nevertheless, the variability in responses is something that both the first-order models of Ergodicity Economics and Expected Utility theory struggle to explain, reflecting the difference between an ideal mathematical model and reality. Both models predict stability in behaviour: Ergodicity Economics predicts that agents should always make time-optimal choices, and Expected Utility Theory predicts that risk-averse agents will always make the risk-averse choice. This variability provides an interesting path for future investigations, such as the role of psychology in time-optimal approaches and the impact of time horizons. Nevertheless, the dependence of human decision-making on proximity to ruin supports the idea that time average optimisation may be a more accurate model for human behaviour than expected value optimisation, as seen in other empirical studies on ergodicity economics.

In conclusion, the time average approaches proposed by ergodicity economics have shown in previous studies [2] [3] that they only need to account for non-ergodicity to explain common patterns in human decision-making. Thus far, the elegant idea underlying ergodicity economics remained limited to a number of dynamic settings. This study opens up this perspective to a new set of dynamics, i.e. additive dynamics in which absorbing states, such as ruin, exist and demonstrates its validity using an experiment with human participants. Our findings further support the notion that humans are sensitive to deviations from ergodicity, be it in the variations of wealth or the evolution of wealth itself. These findings bolster the argument that models that overlook non-ergodicity, like many in traditional economic models, may not accurately capture human decision-making processes and overemphasize their irrationality. This research adds to the expanding collection of studies examining the role of non-ergodicity in economic theories. Numerous economic models rely on expected values, such as models on collaboration [5] and insurance [35], which are subject matter studied both within ergodic economics and by other teams utilizing physics-based methods [21–25] to accurately depict the intricate dynamics inherent in economic exchanges.

Ethics. Prior to the initiation of the study, we sought and received ethical approval from the Ethics Committee for Human Sciences of the Vrije Universiteit Brussel (ECHW). The ethical approval reference number for our study is ECHW_356.02. Before starting the experiment, respondents gave their informed consent to the authors.

Data Accessibility. The computer code and data have been made available at [36]

Authors' Contributions. A.V. and V.G. conceived the experiment, A.V. conducted the experiment and analysis, A.V. and V.G. analysed the results. A.V. and V.G. wrote the manuscript.

Competing Interests. The authors declare that they have no competing interests.

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Appendix

(a) Dyck-Paths

A Dyck path is a unique staircase-shaped walk in the first quadrant of the xy-plane, typically from $(0,0)$ to (n,n), n a natural number or zero. For our purposes, a favourable coin flip equates to making a step of size one to the right, increasing the x-coordinate of the current path. An unfavourable coin flip equates to a step with size one up, increasing the y-coordinate. Let us also assume, for now, that the walk starts at $(0,0)$, which is equivalent to starting with no money. The walk needs to end at (n,n), all the while avoiding ruin. Avoiding ruin entails that the unfavourable flips never outweigh the favourable flips or that the walk never crosses the $y=x$ diagonal. The number of Dyck-paths up to an arbitrary point (n,n) is a bijection of the number of paths that experience ruin for the first time in the point $(n,n+1)$. As a clarification, there are k paths up to a certain point (n,n) that have never crossed the diagonal $y=x$ and, as such, were not (yet) absorbed by the absorbing barrier. From this point, the process can branch in two ways, increasing the total amount of possible paths to $2k$. Of these $2k$ paths, k paths go right and stay alive, and k go up and experience ruin for the first time. In the case of symmetric payoffs, these paths are counted by the simple Catalan numbers C_n, which allows us to quantify the number of paths that encounter ruin for the first time after $2n+1$ coin tosses. The probability of encountering such a path follows from the amount of coin tosses leading up to that point, $\frac{1}{2^{2n+1}}$. This reasoning results in a risk of encountering ruin at $2n+1$ of $\frac{C_n}{2^{2n+1}}$, or a total risk of encountering ruin, given an amount of repetitions t, of $\sum_{n=0}^{t/2-1} \frac{C_n}{2^{2n+1}}$. We can further relax the model to include a starting capital (K) by swapping the Catalan numbers with the Fuss-Catalan numbers [37] $A_n(p,r)$ in our equation. The Fuss-Catalan numbers count the number of walks from $(0,0)$ to (n,n) strictly below the line $y=(p-1)x+r : p \in \mathbb{Z}$ for which $r = \left\lceil \frac{K}{\|\beta\|} \right\rceil$ is wealth rescaled by the magnitude of the negative outcome (β). This entails that for a path to be considered valid, the path should never touch $y=(p-1)x+r$ except in (n,n). Put simply, you are ruined once you reach a capital less or equal to zero if the absorbing barrier is not on zero, one need only rescale r further to accurately depict its stepwise distance from the absorbing barrier.

(b) Bet Couples

Couple(i)	Bet (Riskier)	Bet (Safer)	Couple(i)	Bet (Riskier)	Bet (Safer)	Couple(i)	Bet (Right)	Bet (Wrong)
1	-493	194	**16**	500	-206	**31***	200	200
	485	-202		-506	200		-180	-200
2	490	-202	**17**	-506	204	**32***	190	-202
	-496	196		510	-200		-202	150
3	480	192	**18**	-481	194	**33***	-200	120
	-488	-200		485	-190		210	-200
4	515	-210	**19**	-502	208	**34***	192	192
	-519	206		520	-190		-200	-230
5	495	-198	**20**	505	202	**35***	-204	-204
	-495	198		-511	-208		250	230
6	-506	204	**21**	-518	208			
	510	-200		520	-206			
7	-496	196	**22**	485	-196			
	490	-202		-487	194			
8	-501	206	**23**	-485	190			
	515	-192		475	-200			
9	-499	-202	**24**	520	-202			
	495	198		-514	208			
10	-508	-202	**25**	510	-202			
	510	204		-508	204			
11	-515	206	**26**	-513	202			
	515	-206		505	-210			
12	510	-194	**27**	-507	206			
	-500	204		515	-198			
13	-502	-202	**28**	-485	-194			
	500	200		485	194			
14	480	192	**29**	-493	194			
	-490	-202		485	-202			
15	-513	-198	**30**	-488	196			
	525	210		490	-194			

Table 1: All 35 bet couples used as well as the attention-check bets 31-35. Every bet consists of two outcomes with an equal likelihood of 50/50. Bet couples were shown sequentially to all respondents, and the bet order within a couple was randomised.